JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 21, No. 4, October–December 2007

Technical Notes

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Experimental Investigations of Natural Convection from Circular Plates at Variable Inclination

Ewa Radziemska* and Witold M. Lewandowski†
Gdansk University of Technology, 80-952 Gdańsk, Poland

DOI: 10.2514/1.29027

Nomenclature

C	=	constant in the $Nu(Ra)$ dependence (1)
c_p	=	specific heat of the fluid under constant pressure, $\mathrm{J/(kg \cdot K)}$
D	=	plate diameter, m
d	=	thickness, m
g	=	gravitational acceleration, $\mathrm{m/s^2}$
h	=	heat transfer coefficient, $\mathrm{W/(m^2 \cdot K)}$
$K = k_p/d_p$	=	thermal conductance, $\mathrm{W/(m^2 \cdot K)}$
k	=	thermal conductivity of the fluid, $\mathrm{W/(m \cdot K)}$
k_p	=	thermal conductivity of the plate, $\mathrm{W/(m \cdot K)}$
k_w	=	thermal conductivity of the water, $\mathrm{W/(m \cdot K)}$
$Nu = h \cdot D/k$	=	Nusselt number
n	=	exponent in Eq. (1)
$Pr = \nu/\alpha$	=	Prandtl number
Q	=	heat transfer rate, W
R	=	plate radius, m
$Ra = g\beta\Delta T D^3/(\nu a)$	=	Rayleigh number
T	=	temperature, K
T_c	=	cooler temperature, K
T_w, T_1, T_2	=	wall temperatures, K
T_∞	=	bulk fluid temperature, K
$\alpha = k/c_p \cdot \rho$	=	thermal diffusivity, $\mathrm{m^2/s}$
β	=	average volumetric thermal expansion coefficient
ΔT	=	temperature difference between the plate and the undisturbed region, K
ν	=	kinematic viscosity, $\mathrm{m^2/s}$
ρ	=	density, $\mathrm{kg/m^3}$
ϕ	=	angle of the plate inclination; $\phi = 0$ deg—vertical plate, $\phi = 90$ deg—horizontal plate

Subscripts

c	=	cooler
p	=	plate
w	=	water
1, 2	=	two circular copper plates [Fig. 5, Eq. (2)]
∞	=	undisturbed region

Introduction

HEAT transfer from circular disk geometry is very important in many applications. There has been some experimental research devoted to natural convection heat transfer from stationary circular disks and plates. For a horizontal plate, Kadambi and Drake [1] proposed the correlation for experimental results obtained in air. Faw and Dullforce [2] made interferometric studies for a round plate in air, and Schulenberg [3] proposed similarity solutions for a region around the stagnation point on the isothermal circular plate with specified temperature and specified heat flux. Fujii et al. [4] presented a theoretical study of natural convection heat transfer from downward-facing horizontal surfaces with uniform heat flux. For a vertical plate, Lewandowski and Kubski [5] obtained the mean correlation from the results of 25 researchers. In earlier research [6,7] the present authors presented the results of theoretical and experimental investigations for round horizontal plates facing downward and for vertical ones. Warneford and Fussey [8] made experiments in air for a plate inclined at 85 deg from the vertical.

In this paper an experimental study of natural convective heat transfer from an isothermal round plate facing up at arbitrary angles of inclination has been carried out.

Experimental Apparatus

The experimental apparatus used in the present research consists of the experimental section and the data acquisition system (Fig. 1).

The main part of the experimental section was a specially prepared round copper plate of diameter $D = 0.07$ m immersed in the distilled water (Fig. 2).

The plate had a special layered construction, consisting of two circular copper plates of diameter $D = 0.07$ m, both cemented to a layer of glass laminate using epoxy resin. The plate was electrically heated by the coil powered by a power supply adaptor, which was in contact with one of the copper plates and the plate temperature was regulated. The thickness of the copper plates (5 mm each) ensured that the surface in contact with the surroundings was isothermal (± 0.1 K). The plate and heater were insulated on the side and bottom using polyurethane foam.

A plate was suspended vertically and turned by hand around the axis in a well-insulated glass tank 0.5 m wide, 0.5 m long, and 1.0 m high (Fig. 3).

Calibration of the Plate

At the beginning of the experimental examination the calibration of the plate was performed with the use of the special setup (Figs. 4 and 5.

The tested plate facing downward was located on a Plexiglas ring of height d_w filled with distilled water. During the calibration the heat

Received 27 November 2006; revision received 20 April 2007; accepted for publication 22 April 2007.

*Assistant Professor, Chemical Faculty, G. Narutowicza 11/12, 80-952 Gdansk; ewarad@chem.pg.gda.pl.

†Professor, Chemical Faculty, G. Narutowicza 11/12, 80-952 Gdansk.

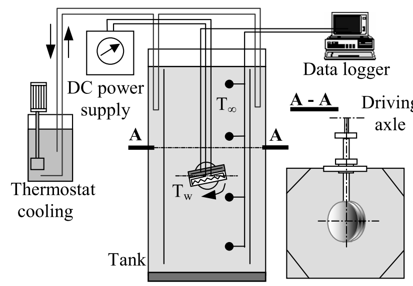

Fig. 1 The schematic of the entire experimental apparatus.

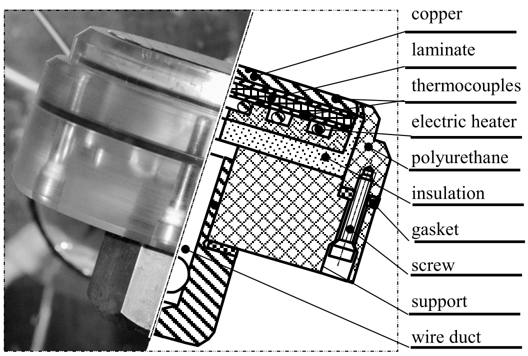

Fig. 2 The view and the cross section of the examined plate.

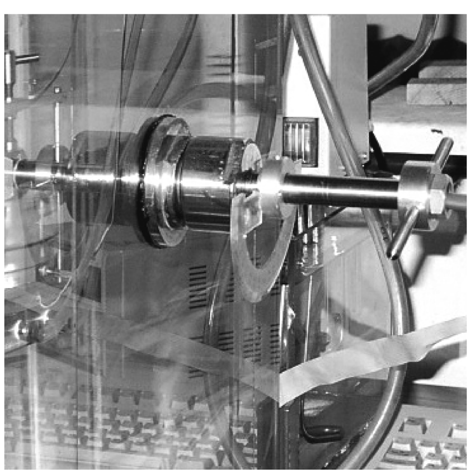

Fig. 3 The plate suspended vertically in a glass tank with manual aligning mechanism.

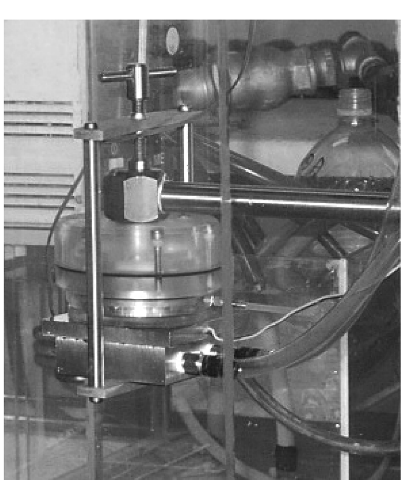

Fig. 4 The view of the special setup for the calibration of the plate.

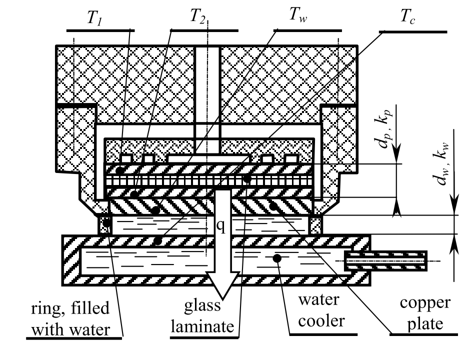

Fig. 5 The special setup for the calibration of the plate.

stream q from the heater of the plate was transported through the plate and water inside the gap of the ring to the cooler.

The reverse configuration eliminates the convective mechanism of heat transfer below the tested plate inside the water. Hence the heat is transferred through the water layer with the thickness d_w only by conduction:

$$Q = \frac{\pi D^2}{4} \cdot \frac{k_w}{d_w} \cdot (T_w - T_c) \tag{1}$$

This heat transfer rate is equal to the heat transfer rate through the measured plate (two on Fig. 1), consisting of the glass laminate and two copper plates:

$$Q = \frac{\pi D^2}{4} \cdot K \cdot (T_1 - T_2) \tag{2}$$

where K is the thermal conductance of the measured plate and $T_1 - T_2$ is the temperature difference between the two copper plates of the measured plate.

The relation describing the thermal conductance K of the plate as a function of the average temperature of the plate $\bar{T}_p = (T_1 + T_2)/2$ was determined (Fig. 6). The uncertainty in the determination of K was estimated to be $\pm 5.0\%$.

Eleven copper-constantan thermocouples were attached to each side of the test plate and placed in the undisturbed region. A computer program processed the output signals from the thermocouples. The average temperature of the plate was the average of the four temperatures measured on that plate. Quantitative investigations of natural convective heat transfer were carried out for different temperatures of the heated surface.

The heat transfer rate through the plate is equal to the heat transfer rate from the plate to the fluid. From Newton's law it can be calculated as

$$Q = \frac{\pi D^2}{4} \cdot K(\bar{T}_p) \cdot \Delta T_p = \frac{\pi D^2}{4} \cdot h \cdot \Delta T \tag{3}$$

The Nusselt number was then calculated as

$$Nu = K\left(\bar{T}_p\right) \cdot \frac{\Delta T_p}{\Delta T} \cdot \frac{D}{k} \tag{4}$$

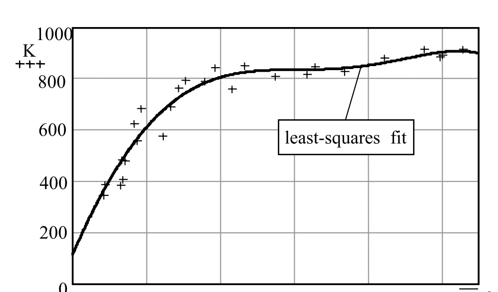

Fig. 6 Thermal conductance *K* of the tested plate in the function of the mean plate temperature.

To minimize the influences of thermal property variations on the measurements, the ambient temperature of the water was maintained at the stable value of 20°C.

Thermophysical properties of the examined fluid were estimated at the temperature defined as the mean temperature of $(T_w + T_\infty)/2$.

The Nusselt number was determined with a maximum error of 6.7%, and the Rayleigh number was determined with a maximum error of 4.5%.

Results

The aim of this work was to obtain the empirical correlation for natural heat transfer from the plates at arbitrary angles of inclination in the classical form [9]:

$$Nu = C \cdot Ra^n \tag{5}$$

where, in this particular case, can be written as [10]

$$Nu = C \cdot (Ra \cdot \cos\phi)^n \tag{6}$$

where the Nusselt and Rayleigh numbers are based on the characteristic linear dimension. In the majority of works, concerning to vertical plates, the plate diameter D is the characteristic linear dimension and the $Nu(Ra)$ correlation exponent n is equal to 0.25. In the case of horizontal plates the plate radius R is chosen as the characteristic linear dimension and the $Nu(Ra)$ correlation exponent n is equal to 0.2. In the present investigations the Nusselt and Rayleigh numbers are based on the plate diameter D.

Results of experimental investigations obtained for the round plate at different inclinations in water, and for comparison, for the previously measured case of the vertical round plate in air [7] with the correlation $Nu = 0.655 \cdot Ra^{0.25}$ and the vertical plate in water, described by $Nu = 0.587 \cdot Ra^{0.25}$ [7], are presented in the logarithmic Nu and Ra dependence (Fig. 7). All the results concerning arbitrary inclinations were made for similar values of the Rayleigh number to quantify the influence of the inclination angle only. The mean correlation can be written in the form $Nu = 1.0 \cdot Ra^{0.2}$ or $Nu = 0.4 \cdot Ra^{0.25}$.

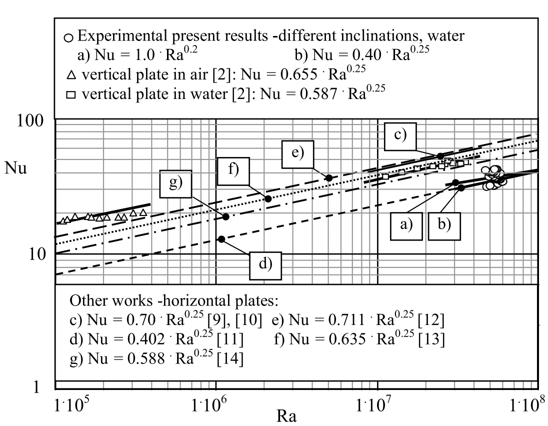

Fig. 7 Experimental of $Nu(Ra)$ dependence for different angles of plate inclination (○) in comparison to other results.

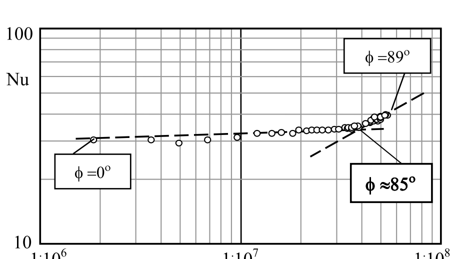

Fig. 8 Experimental $Nu(Ra \cdot \cos\phi)$ dependence for different angles of plate inclination.

The dependence between Nu and $Ra \cdot \cos\phi$ is presented in Fig. 8 in the logarithmic scale. It could be found that the impact of the plate's slope is visible for configurations close to the horizontal position and the transient point can be marked near 85 deg from the vertical position.

Conclusions

Natural convective heat transfer from an isothermally heated upward facing circular plate with variable angle of inclination was experimentally investigated. The comparison of the present results with the previous study of convective heat transfer from a round vertical plate [7] in air and water and also a horizontal plate, in the characteristic case of the inclined plate, as well as the comparison with results of other authors [11–16] confirms positive acknowledgment. This compatibility allows an explanation of the plate inclination influence on convective heat transfer intensity.

The substantial impact of the plate slope was observed for configurations close to a horizontal position. For angles greater than 85 deg (measured from the vertical position), the Nusselt number and the heat transfer coefficient strongly grows with the Rayleigh number. This transient point suggested that a mechanism of separation of the boundary layer from the heated plate and transforming it into a buoyant plum is similar to what was observed in the case of the rectangle isothermal slightly inclined from the horizontal position plates [17]. However the confirmation of this needs to perform the next study especially with the use of visualization methods of convective fluid flow patterns over round inclined plates. Such visual investigations as well as the qualitative investigations of heat transfer from the isothermally heated circular plate facing upward at arbitrary angles of inclination should be a subject of future research.

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